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Structural stability and Goodwin's growth cycle

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Abstract

The methodological prescription that a structurally unstable model should be rejected is critically analysed by considering Goodwin's predator–prey model [GPPM; Goodwin, R.M., 1967. A growth cycle. In: Feinstein, C.H. (Ed.), Socialism, Capitalism and Economic Growth. Cambridge University Press, Cambridge; Goodwin, R.M., 1972. A growth cycle. In: Hunt, E.K., Schwartz, J.G. (Eds.), A Critique of Economic Theory. Penguin, Harmondsworth]. It is argued that structural instability is not sufficient to reject GPPM and that structurally stable extensions yielding limit-cycles are not necessarily more appropriate formalisations of distributive conflict, from both a theoretical and an empirical viewpoint. An interpretation of GPPM and of the empirical evidence is proposed, which takes into account structural instability.

JEL classification: B41; C62; E32; J20

Keywords: Dynamic systems; Structural instability; Distributive conflict; Cycles

1. Introduction

The concept of structural stability has been formalised with the tools of modern topology relatively recently by Andronov and Pontryagin (1937).[1] Unlike dynamic stability, which relates to a property of a state or an orbit of a dynamic system, structural stability relates to a property of a dynamic system itself. It incorporates the idea that if the system is slightly perturbed, the qualitative features of the phase portrait do not change in an essential way.

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[1] For thorough discussion of the origins of the concept of structural instability and its evolution in the philosophy of sciences, see Vercelli (1991).

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In the received view, structural stability represents a fundamental criterion in model building. It is considered a necessary condition for the observability and predictability of scientific phenomena, and a structurally unstable model is deemed both inherently unable to describe an empirical regularity and inadequate for predictive purposes. "It would be infinitely improbable to observe the existence of unstable structures because they are transient and unobservable" (Vercelli, 1991, p. 51).[2]

The methodological prescription that a structurally unstable model should be rejected has recently been called into question. For instance, in economic theory, rational expectation macroeconomics has rejected the 'stability dogma' (Gandolfo, 1997, p. 338) – albeit with no explicit methodological discussion – by adopting models characterised by structurally unstable saddle point dynamics (George and Oxley, 1985; Vercelli, 1991). More generally, Vercelli (1991) has challenged the received view from a broad methodological perspective, arguing that structural instability is not sufficient to reject a model a priori and suggesting the need for a less dogmatic approach to model building. Yet, despite increasing scepticism (see also Gandolfo, 1997, pp. 338–341; Gabisch and Lorenz, 1989, Section 6.3), the 'stability dogma' remains part of the received view on model building.

In this paper, the methodological prescription that a structurally unstable model should be rejected is not discussed from a general perspective. Instead, the received view on the inadequacy of structurally unstable models in economics, and the main methodological issues concerned, are discussed by analysing the specific case of Richard Goodwin's predator–prey model (GPPM; Goodwin, 1967, 1972).

GPPM is one of the first and most elegant dynamic formalisations of Marx's theory of distributive conflict and a seminal contribution in the use of non-linear models drawn from mathematical biology to analyse economic phenomena—in particular, growth cycles. A large number of authors have explored the positive and normative properties of the model. Yet in GPPM the cyclical interaction between employment and the workers' share of income is modelled by a structurally unstable Lotka–Volterra dynamic system (LV). In most (albeit not all; see note 11) of the literature, this is considered as the "main defect" (Medio, 1979, p. 17), a major drawback (Sato, 1985, p. 21; De Nicolò, 1988, p. 64), the "worst fault of the system" (Farkas and Kotsis, 1992, p. 514), an "undesirable quality for economic models" (Sportelli, 1995, p. 37), the main reason why "the Lotka–Volterra cycle... is not a 'good' fluctuation model" (Manfredi and Fanti, 2004, p. 574). Many of the extensions of GPPM have been explicitly aimed at avoiding structural instability, based on the view that if it "is to provide a satisfactory model of economic cycles, changes must be made to its mathematical structure so that the governing differential equations exhibit stable limit cycle behaviour" (Chiarella, 1990, p. 70).

The main aim of this paper is to evaluate this view critically. First, it is argued that structural instability should not be considered sufficient to reject GPPM a priori. A different methodological approach is advocated, based on the analysis of the empirical features of the phenomenon investigated and the main characteristics of the theory that the model is meant to formalise. From both points of view, GPPM may provide a satisfactory formalisation of Marx's theory of distributive conflict, even as compared to its structurally stable extensions yielding limit cycles. In order to support this view, an interpretation of GPPM that takes into account structural instability is proposed. It is argued that GPPM isolates in vitro the main forces underlying distributive conflict in

[2] The similarity with Samuelson (1947) argument concerning dynamic instability is obvious.

the short-run, but it may be misleading to use it simply to *describe* the data pattern when those data are subject to perturbations.

The structure of the paper is as follows. In Section 2, GPPM is presented and the topological concept of structural stability is discussed. Then the attempts at a microfoundation of the structurally unstable specification of GPPM are critically reviewed. In Section 3, GPPM and structurally stable extensions are compared as alternative formalisations of distributive conflict. In Section 4, a different interpretation of GPPM based on its structural instability is provided, rooted in the empirical evidence on Goodwin cycles. Section 5 is devoted to conclusions.

2. Goodwin's model and structural instability

Let s denote savings, k capital, q output, i investment, l employment, and w the real wage rate. The assumptions of Goodwin (1967, 1972) model can be summarised as follows:[3]

(A1) There is a fixed capital–output ratio, $\sigma \equiv k/q$, so that $\dot{k}/k = \dot{q}/q$.

(A2) It is assumed that Say's law holds, profits are completely reinvested and workers do not save. Formally: $s = i = \dot{k} = q - wl = q[1 - (wl/q)]$.

Let $u \equiv wl/q$ be the share of labour in national income. Dividing the formula in (A2) by k and using (A1), one obtains $\dot{k}/k = q(1-u)/k = (1-u)/\sigma$.

(A3) Exogenous, constant growth rates of productivity, $a \equiv q/l$, and of population, n, so that $a = a_0\, e^{\alpha t}$ and $n = n_0\, e^{\beta t}$. Thus $\dot{q}/q = \dot{l}/l + \alpha$.

Let $v \equiv l/n$ be the rate of employment. After some algebra, from (A1) to (A3):

$$\frac{\dot{v}}{v} = \left[\frac{1}{\sigma} - (\alpha + \beta)\right] - \frac{u}{\sigma}. \tag{1}$$

(A4) The real wage bargaining equation is

$$\frac{\dot{w}}{w} = -\gamma + \rho v,$$

where $\gamma, \rho > 0$. Hence, given that $u = w/a$:

$$\frac{\dot{u}}{u} = -(\gamma + \alpha) + \rho v. \tag{2}$$

Eqs. (1)–(2) form a LV dynamic system that is a special case of the general dynamic systems:

$$\frac{\dot{v}}{v} = F(v, u), \tag{3}$$

$$\frac{\dot{u}}{u} = G(u, v), \tag{4}$$

where F and G are linear and the zero restrictions on the parameters yield a LV model displaying conservative oscillations around a neutrally stable equilibrium, the *centre* (Velupillai, 1979;

[3] For a detailed presentation of GPPM, see e.g. Desai (1973), Blatt (1983), Gandolfo (1997).

Flaschel, 1984; Kruger, 1985). Starting from any initial point – except the centre – as employment grows, workers' bargaining strength increases, yielding an increase in the growth rate of real wages, which cuts into profits thereby reducing accumulation. As accumulation slackens, the employment rate decreases, thus decreasing the growth rate of real wages and restoring the profitability conditions necessary for accumulation. This mechanism – which repeats itself forever – incorporates a Marxian view of the fundamental dynamic contradictions of capitalist economies that arise from class conflict.[4]

Despite its simple structure, GPPM has many interesting features that have been extensively analysed in the literature—such as the possibility of modelling cycles and growth together, its non-linear structure, etc. This paper focuses on the structural instability of (1) and (2), a feature that raises important methodological issues and has led many authors to question the model.

Structural stability can be formally defined as follows. Let $x, y \in \Re^n$ and let $\langle x, y\rangle$ be the inner product of x and y. Let $D^n = \{x \in \Re^n : |x| \leq 1\}$, where $|x|$ is the Euclidean norm of x. Let ∂D^n be the boundary of D^n: $\partial D^n = \{x \in \Re^n : |x| = 1\}$. Let C^1 be the set of bounded and continuous functions on $\Re^n$. Let W be an open set in $\Re^n$ and let $V(W)$ denote the set of all C^1 vector fields on W. The topological definition of structural stability usually adopted in the literature on GPPM (e.g., Velupillai, 1979; Flaschel, 1984) is given by Hirsch and Smale (1974, pp. 312–313).[5]

Definition 1. Consider C^1 vector fields f: $W \to \Re^n$ defined on some open set W containing D^n such that $\langle f(x), x\rangle < 0$ for each $x \in D^n$. f is *structurally stable* on D^n if there is a neighbourhood $I \subset V(W)$ such that if g: $W \to \Re^n$ is in I, then the flows of f and g are topologically equivalent on D^n; that is, there is a homeomorphism h: $D^n \to D^n$ such that $h(\{\phi_t(x) | t \geq 0\}) = \{\psi_t(x) | t \geq 0\}$, for each $x \in D^n$, where $\phi_t(x)$ is the flow of f, ψ_t is the flow of g, and if x is not an equilibrium, h preserves the orientation of the trajectory.[6]

In other words, the topological definition of structural stability incorporates the intuitive idea that the features of the phase portrait do not change in an essential way if the system is infinitesimally perturbed, by requiring that "nearby" dynamic systems have the same qualitative dynamics.

As it is clear from Definition 1, the concept of structural stability is built upon an unambiguous understanding of three primitive notions, viz. "(i) a notion of equivalence of systems; (ii) a topology of systems; (iii) a notion of perturbations which are possible a priori" (Cugno and Montrucchio, 1982, p. 96). However, as argued by Cugno and Montrucchio (1982), their unambiguity is just presumed, whereas all three notions can be interpreted in a number of theoretically relevant ways and each will change the notion of structural stability.[7] As argued by

[4] Although GPPM "conflicts directly with Marx's claims of progressive immiserisation of the workers" (Blatt, 1983, p. 204). Further, as noted by Solow (1990), in GPPM workers play the role of predators, whereas capitalists are the preys. The analogy to predator–prey models in mathematical biology, however, is easily strained, as Solow concedes. The relation between GPPM and Marx is an important and interesting topic, and it is analysed at greater length in a companion paper (Mohun and Veneziani, 2006). See also Asada (1989) and Gabisch and Lorenz (1989).

[5] For an analysis of alternative definitions, see Andronov and Pontryagin (1937), Abraham and Marsden (1980), Kruger (1985), Vercelli (1991), Gandolfo (1997).

[6] The orientation of a trajectory is the direction that points move along the curve as t increases.

[7] For instance, the topological definitions of structural stability normally exclude perturbations that alter the order of the system. However, "the excluded case may be of the utmost scientific interest. This is the case, for example, of the so-called 'parasitic' parameters which play an important role in many fields of physics and engineering" (Vercelli, 1991, p. 48).

Vercelli, "the choice of one definition rather than another depends on opportunity considerations which are related in their turn to the mathematical nature of the problem,... but also to the characteristics of the problem to which the mathematical instruments are applied" (Vercelli, 1982, p. 178).

If the demanding notion of topological equivalence is adopted, as in Definition 1, GPPM is not structurally stable: there is a class of infinitesimal perturbations of the coefficients of the reduced form (1) and (2) that alter its qualitative features. In fact, the centre is a non-hyperbolic equilibrium, i.e., the Jacobian of the linearised system around the centre has eigenvalues with real part equal to zero. Hence, an infinitesimal perturbation that leads to a non-zero trace of the Jacobian can make it into a sink or a source. It is not true, however, that "every minimal alteration of the structure of GPPM would destroy its main characteristic" (Medio, 1979, p. 33), the closed orbits. Even if topological equivalence is adopted, GPPM is structurally stable with respect to a class of perturbations of the Phillips curve (Cugno and Montrucchio, 1982, pp. 96–99; Velupillai, 1979; Kruger, 1985). In general, it is untrue that GPPM "is not suitable for any extension due to its structural instability" (Medio, 1979, p. 33). The closed orbits are preserved by extensions that include nominal wages, an independent investment function, and a monetary sector (Izzo, 1971; Di Matteo, 1984; Asada, 1989); an endogenous labour supply, capital depreciation, wealth effects on savings, and workers savings (van der Ploeg, 1984); taxation and unemployment benefits with a balanced budget (Glombowski and Kruger, 1984); inflation, the foreign sector, and exchange rate dynamics (Pugno, 1998); a non-linear wage bargaining equation and a non-linear investment function (Desai et al., in press).

However, much weaker than topological equivalence is a notion of observational, or practical, equivalence which Cugno and Montrucchio (1982, p. 99) argue may be more relevant to observational sciences such as economics. This only requires the perturbed orbits to remain in a neighbourhood of the orbits of the original system for a sufficiently long time (where the length would depend on the problem studied). From this perspective, a conservative system and a dissipative system with 'very slow' convergence to the equilibrium may be considered as equivalent for all practical purposes. By using the notion of observational equivalence Cugno and Montrucchio (1982, p. 99) prove the structural stability of GPPM under even more general perturbations.

In the case of GPPM, the inherent ambiguities in the formal definition of structural stability are also reflected in the difficulty of providing a fully satisfactory proof of its structural instability. First, consider Velupillai (1979) attempt at applying the structural stability theorem, which characterises structurally stable systems in the plane. The theorem can be stated as follows.

Theorem (Hirsch and Smale, 1974, p. 314). "Let f: $W \to \Re^2$ be a C^1 vector field on an open set $W \supset D^2$ such that f points inward along the boundary ∂D^2 of D^2, i.e. $\langle f(x), x\rangle < 0$ if $x \in \partial D^2$. Then f is structurally stable on D^2 if and only if (a) the equilibria in D^2 are hyperbolic and (b) each closed orbit in D^2 is either a periodic attractor or a periodic repeller and (c) no trajectory in D^2 goes from saddle to saddle".

Velupillai (1979) proves that condition (a) is violated, but as shown by Flaschel (1984) and Kruger (1985), GPPM does not lie in the restricted set of vector fields considered in the theorem because f does not point inward along ∂D^2 (as noted also by Desai et al. (in press), who have extended GPPM in order to constrain cycles to be in the unit box), and thus the theorem cannot be applied.

Second, the alternative proofs provided – such as Desai (1973), van der Ploeg (1983), Flaschel (1984), Asada (1989, pp. 156ff), Chiarella (1990), Choi (1995), Aguiar (2001) – that aim to replace

"Velupillai's purely formal and insufficient proof by simple economic reasoning" (Flaschel, 1984, p. 63), are not entirely convincing from a methodological viewpoint. In these papers, GPPM is shown to correspond to a bifurcation value of the parameters of a more general model in which it is embedded, and if the parameters are slightly perturbed, the dynamics of the system radically changes.[8] These proofs are intuitively appealing, but their methodological foundations are unclear: without a precise prescription concerning the extensions allowed, it is not clear that this type of argument can discriminate between stable and unstable models. In principle, by choosing the appropriate generalisation, the structural instability of virtually any model could be proved, especially if extensions altering the order of the system were included. As is well known, the "likelihood" of structural stability decreases as the order of the system increases so that "even the genericity of structural stability in the compact manifold of dimension 2 would be jeopardized" (Vercelli, 1991, p. 48) by perturbations that alter the dimension of the system.

These observations are not meant to deny the structural instability of GPPM – as noted earlier, the features of the phase portrait are indeed sensitive to a class of infinitesimal perturbations – but only to cast it in a different light. They suggest that in general, "the query of whether or not a certain system... is structurally unstable cannot be answered with a simple yes or no" (Vercelli, 1982, p. 179)[9]; and they should raise doubts about a mechanical application of the methodological prescription that a structurally unstable model should be rejected.

Before turning to an analysis of GPPM that takes account of its structural instability, it is opportune to consider a methodological argument often used to defend structurally unstable models in economics. As noted above, new-classical macroeconomics has tacitly reversed the received view on structural instability by focusing on unstable saddle-path equilibria. In Ramsey–Cass–Koopmans models, this is possible thanks to the assumption that jump variables are chosen so as to place the economy on the saddle-path; that is, "an ad hoc assumption is brought into play to change a structurally unstable outcome to a structurally stable one by restricting the class of solution paths to the stable branch alone" (George and Oxley, 1985, p. 312). Similarly, if microfoundations were provided to GPPM, it might be argued that structural instability is not a major problem, since agents choose the zero restrictions on (3) and (4) yielding the LV specification. Thus, instability "is only a mathematical aspect, from an economic point of view the model is stable: the competition between workers and capitalists leads to closed orbits" (Ricci, 1988, p. 53). The rest of this section aims to show that the attempts to provide microfoundations to GPPM are not convincing, so that the argument cannot be invoked and the issue of structural instability cannot be avoided.

Balducci, Candela and Ricci attempt to establish "conditions of robustness for Goodwin dynamic properties" (Balducci et al., 1984, p. 50) by differential games' tools. In their model, fully unionised workers choose the intensity of their wage claims, say ρ^{w}, and fully organised capitalists choose the share of their profits, say h^{c}, to be invested (i.e. (A2) above does not hold). Assuming diminishing marginal utility of consumption, both classes prefer orbits characterised by smaller amplitude, the centre being the preferred orbit. Hence, classes are assumed to choose their strategies to minimise a quadratic loss function that depends on the difference between actual

[8] The concept of *bifurcation* is related to that of structural stability, since at a *bifurcation* point a system discontinuously changes its type of motion (see e.g. Gabisch and Lorenz, 1989).

[9] Moreover, a system may be structurally stable with respect to infinitesimally small perturbations but not with respect to perturbations of a small but finite magnitude. Hence, it would be more suitable to talk about "a degree of structural instability" (Vercelli, 1991, p. 48). This issue is not discussed here since GPPM is unstable with respect to a class of infinitesimal perturbations.

and desired parameters, subject to the following equations describing the state dynamics:

$$\frac{\dot{v}}{v} = \left[\frac{1}{\sigma} - (\alpha + \beta)\right] - \frac{h^{c}u}{\sigma}, \tag{1'}$$

$$\frac{\dot{u}}{u} = -(\gamma + \alpha) + \rho^{w}v. \tag{2'}$$

They prove that GPPM coincides with the steady state equilibrium of a non-cooperative Nash game. Hence, they conclude that "the periodic and conservative fluctuation of Goodwin's dynamics is not dependent on irrational behaviors of players, but is rather a consequence of the conflicting elements proper of a pure capitalism" (Balducci et al., 1984, p. 64).

In a similar attempt, Mehrling (1986) assumes that capitalists choose the level of investment to maximise the present value of their profits subject to the constraint that investment cannot be negative and cannot exceed capitalists' profit share. In turn, workers choose the rate of wage change to maximise the present value of their consumption, subject to an upper bound to their demands as in A4:

$$\frac{\dot{w}}{w} \leq -\gamma + \rho v. \tag{5}$$

The analysis of the economy under different institutional structures shows that, when both classes are unorganised, GPPM arises as an equilibrium of the game, with profits being completely reinvested and (5) binding.

Neither attempt, however, seems to provide satisfactory microfoundations to the structurally unstable specification of GPPM. By assuming that (1′) and (2′) describe the motion of the states and by focusing only on steady states, Balducci et al. (1984) effectively assume the problem away. Moreover, it should be noted that the steady-state equilibrium "is not globally asymptotically stable" (Balducci et al., 1984, p. 58) and thus it is not obvious that the economy will actually approach Goodwin's specification. In Mehrling (1986), too, constraint (5) remains unexplained. Besides, even assuming, for the sake of the argument, that the model effectively characterises the institutional setting in which Goodwin's parameters emerge as a rational choice of the two classes, Mehrling proves that if workers are unorganised, capitalists can improve by coordinating their actions, and vice versa. Therefore both classes have a strong incentive to abandon their atomistic behaviour, and the institutional structure yielding GPPM is not robust.

In sum, given the difficulty of providing sound microfoundations to GPPM, it is not possible currently to argue that structural instability is irrelevant because agents choose Goodwin's parameters. Thus the methodological issue regarding the wisdom of adopting a structurally unstable model has to be addressed.

3. Structural instability and distributive conflict

Section 2 shows several conceptual problems with the notion of structural stability. Yet, even setting these issues aside, the methodological prescription that a structurally unstable model should be rejected a priori is not compelling. As argued by Vercelli (1991), structural stability is not necessary to guarantee the predictability or observability of empirical phenomena, and the "stability dogma" seems based on the a priori conviction that reality is indeed stable. "Modern physics, however, is rich in experimental examples of structurally unstable systems, which suggest a revision of fundamental attitudes toward reality" (Gabisch and Lorenz, 1989, p. 203). Relevant examples are the field of synergetics and the thermodynamic theory of dissipative structures; but

Hamiltonian models in mechanics, too, are structurally unstable. Besides, in the light of the failure to prove the genericity of structural stability for systems of dimension greater than two, there is no reason to believe that complex economies would necessarily be structurally stable, or that systems characterised by some degree of instability are uninteresting or residual. This suggests that the adoption of a structurally unstable model should be evaluated on a case-by-case basis, depending on the nature of the phenomenon investigated and the aims of the research.[10]

In most of the literature on GPPM,[11] however, structural instability is considered a major shortcoming of the model and several extensions of GPPM are explicitly aimed at yielding a structurally stable limit cycle. For example, Medio (1979) introduces effective demand issues and assumes firms adopt mark-up pricing, given money wage bargaining; Sato (1985) assumes the economy is divided into two sectors and incorporates a full-employment constraint and a real wage constraint in a discrete time version of GPPM; De Nicolò (1988) assumes technical progress in the form of learning-by-doing and variable capacity utilisation; Chiarella (1990) introduces a time lag in the wage bargaining equation, due to sluggish wage adjustment; Farkas and Kotsis (1992) also modify the wage bargaining equation, introducing time lags and assuming logistic saturation, instead of exponential growth, at a zero real wage; Sportelli (1995) adopts an investment function including profit expectations; Fanti and Manfredi (1998) analyse profit sharing rules; Manfredi and Fanti (1999) consider gestation lags and efficiency wages.[12]

To be sure, many factors are important in the evaluation of a model, such as the plausibility of the assumptions, its predictive and explanatory power, etc. Some of the extensions of GPPM can be deemed more satisfactory than the original model according to these criteria. This section focuses, however, on stability properties as one important element in the evaluation of a model. In particular, it is argued that structural instability does not make GPPM an unsatisfactory model of the basic mechanism underlying class conflict – and in particular of Marx's theory – as compared to extensions yielding limit cycles, if the characteristics of the theory that the model is meant to formalise and the features of the phenomenon investigated are considered.

Concerning the former issue, models yielding limit cycles arguably suggest a reductionist, if not mechanistic, view of distributive conflict, and in particular of Marx's theory. Indeed, in his seminal paper on LV systems, Samuelson (1972) rejects the structurally unstable specification of LV and, by setting up what might be considered the prototype of the models of conflict yielding limit cycles, he obtains "a fundamental cyclical mechanism which could operate almost independently of the institutional environment... If one posits two permanent features of economic life – exploitation and increasing returns – one can deduce a permanent business cycle (and moreover, one with a unique amplitude of fluctuations as well as period)" (Samuelson, 1972, p. 473). This view seems at odds with Marx's picture of the dynamic nature of capitalist economies.

Instead, consider an interpretation of Marx's analysis of capitalism based on the notions of structural change and structural instability.[13] While class conflict is inherent in capitalist societies, how this conflict plays itself out tends to change over time, in a potentially discontinuous fashion,

[10] For instance, a structurally unstable model may be necessary to understand the stabilisation process of an economy (Vercelli, 1991, p. 53). It should be noted that the trade-offs between different characteristics of alternative models are important, too: for instance, one may opt for a structurally unstable model if it is computationally efficient vis-à-vis a structurally stable one.

[11] With significant exceptions, such as Cugno and Montrucchio (1982), Vercelli (1982), and Kruger (1985). See also Velupillai (1982, p. 81, fn. 2): "I am myself guilty of attaching a great deal of importance to [structural stability]. I am, now, less convinced about the importance and necessity of structural stability".

[12] See also van der Ploeg (1983), Flaschel (1988), Skott (1989), Flaschel and Groh (1995).

[13] For an analysis of Marx's theory based on structural instability, see Vercelli (1984).

due to endogenous forces that modify the balance of power between classes and the structure of the bargaining process (Marx, 1954, pp. 596–597). GPPM could then be seen as a stylised, in vitro portrait of the basic forces underlying class conflict, in which its structural instability reflects the posited fragility of the structure of the symbiotic mechanism regulating distributive conflict.

Concerning the empirical evidence, the available studies do not show the "fundamental cyclical mechanism" that would be implied by a limit cycle. In his analysis of the US post-war uv-plot, Solow identifies both "a suggestion of predominantly clockwise motion, but in three separate episodes" (Solow, 1990, p. 39); and "a bare hint of a single large long-period clockwise sweep" (Solow, 1990, p. 40). Flaschel and Groh (1995) and Harvie (2000) find a similar pattern in the post-war (u, v) plots of, respectively, 8 and 10, OECD countries, with shorter (10–15 years) clockwise cycles around an incomplete long-period (50–70 years) clockwise motion. Hence, if anything, the "fundamental cyclical mechanism" represented by a limit cycle should describe the long run motions. Indeed, authors such as Farkas and Kotsis (1992) or Flaschel and Groh (1995), who have adopted a structurally stable generalisation of GPPM in their empirical work, have opted for an "approach which attempts to argue for a large phase length and amplitude of this growth cycle" (Flaschel and Groh, 1995, p. 295).[14] From this perspective, the analysis should focus on a long run conflict between workers and capitalists, influenced by secular forces such that any shorter-run factors are averaged out.

This interpretation is not entirely convincing. It is unclear that there is a long run empirical regularity to be explained in the first place. In several countries the long run motion is not well shaped. Even in the USA, Mohun and Veneziani (2006) show that if Goodwin's variables are accurately defined and long-run and short-run motions are properly distinguished, the existence of a long-run cyclical motion is questionable. In any case, "one cycle is not a periodic motion" (Solow, 1990, p. 40) and much less so the "three-quarter cycle" (Harvie, 2000, p. 357) that emerges from the plots of the countries considered. In order to support a long run interpretation, one would need much longer time series than those considered by Farkas and Kotsis (1992), Flaschel and Groh (1995), Harvie (2000). When longer time series are analysed, however, the existence of stable long run cycles is questionable, as shown in the UK 1855–1965 data by the "steady shift to the right (rising u)" (Desai, 1984, p. 259) of the uv-trajectories.

Besides, if GPPM and its extensions are meant to formalise Marx's theory of the "industrial reserve army", a long run interpretation contradicts Marx's own emphasis on a cyclical mechanism definitely shorter than 50–70 years (e.g. "a decennial cycle" (Marx, 1954, p. 593 and passim)). Finally, the behavioural foundations of such secular motions are not obvious, especially given the presence of significant structural breaks (see, e.g., Mohun and Veneziani, 2006).[15]

In sum, a structurally stable model yielding a limit cycle is not necessarily more appropriate to formalise Marx's theory of distributive conflict and it "may be too robust to serve as a 'first step' theoretical description of empirically less stable structures" (Kruger, 1985, p. 35). On the contrary, an interpretation of distributive conflict focusing on qualitative change and structural instability seems more in line with Marx's theory and it may be helpful to analyse the data. In

[14] "...[U]nder modern post-war conditions... the large changes in income distribution among capital and labor which this reproduction process calls for will use up an amount of time which can be drastically higher than the 8–10 years that Solow calculated..." (Flaschel and Groh, 1995, p. 294).

[15] Dore (1993, pp. 207–209), too, rejects the long-run analysis based on a short-run interpretation of the mechanism incorporated into the real wage Phillips curve. For an interesting discussion of the period of the growth cycle, see Glombowski and Kruger (1988).

order to support this argument, however, an interpretation of GPPM consistent with the available empirical evidence should be provided.

4. Goodwin's model reconsidered

From a mathematical point of view, LV may be interpreted as the non-linear analogue of the frictionless oscillator: "every physicist knows that this simple harmonic motion is best for explaining what oscillation is and how, why and when it arises. He is also aware that it is never found in practice" (Goodwin, 1984, p. 68). Similarly, it may be argued that Goodwin's "starkly schematized and hence quite unrealistic" (Goodwin, 1967, p. 54) model yielding conservative oscillations offers a stylised, in vitro portrait of the basic mechanism underlying the interaction between classes. It models "the most essential dynamic aspects of capitalism. . . [In order] to show the logic and plausibility of a type of behaviour and of its analysis, it is essential to get it clearly and simply stated" (Goodwin, 1972, p. 443). The basic mechanism depicted by GPPM, however, is subject to perturbations that tend to alter its main qualitative features.

From this perspective, GPPM should be seen as a theoretical model to understand distributive conflict, rather than as a self-contained formal description either of a long run cycle or of a short-run persistent cycle in income shares and employment. It is a stylised, in vitro portrait of the basic forces underlying class conflict, and its structural instability is an extreme picture of the fragility of the structure of the symbiotic mechanism regulating distributive conflict.

If correct, this interpretation has interesting implications for the empirical and theoretical analysis of distributive conflict in a Marx/Goodwin framework. As concerns the former, first, the interpretation is consistent with the fact that empirically, the mechanism described by GPPM may be subject to perturbations, as suggested by the absence of persistent periodic motions in the uv-plots and by the econometric results in Desai (1984) and Harvie (2000). Desai (1984) embeds GPPM into a more general model with money wage bargaining and adaptive expectations, in order to test the parameter restrictions that yield the structurally unstable LV specification. The analysis of the UK data 1855–1965 leads to the rejection of such restrictions and thus of the original specification of GPPM.[16] Harvie (2000) obtains similar results for ten OECD countries in the period 1951–1991. If structural changes dominate the long run motion of (u, v), these results should not be too surprising. As noted by Solow, "the model does capture something real. At least I can think of episodes that seem to conform to that [cyclical] pattern. I think that is all that can be said and all that need be said about a small model. . . That is why, methodologically speaking, small is beautiful" (Solow, 1990, pp. 38–39).

Second, the proposed interpretation of GPPM suggests an alternative explanation of the uv-plots, based on the idea that the Marx/Goodwin cycles are the shorter run cycles that appear around the long run motion (a detailed empirical analysis supporting this interpretation is provided in a companion paper; see Mohun and Veneziani, 2006). The shorter-run cycles tend to become visible in a less perturbed environment, whereas structural change is causing their position to shift.[17] This is consistent with the rejection of the zero restrictions of LV and it may help to

[16] In the UK data 1855–1965, the assumption of constancy of the capital–output ratio is always rejected, while the results on money illusion and adaptive expectations are less clear-cut.

[17] Dore (1993, p. 207) briefly discusses a similar interpretation. Harvie (2000, p. 357) mentions it, too, but he opts for the long-run interpretation of GPPM without a proper discussion. In a very interesting attempt at modelling both short and long run fluctuations, Skott (1993) argues that Goodwin cycles explain the longer-run motion, while Keynesian mechanisms dominate short-run cycles. However, as argued in Section 3, the long-run interpretation of GPPM is not persuasive.

explain at least some of the puzzling econometric results obtained by Harvie (2000) based on the long run interpretation of GPPM, such as the estimated centre lying outside the uv-plot, an estimated 1-year period of the cycle, and rather unrealistic values of the Phillips curve parameters in all countries.[18]

Moreover, the period T of Goodwin's cycles is determined by the model's parameters: with a linear wage bargaining equation, it can be shown (Atkinson, 1969; Harvie, 2000) that around the centre $T = 2\pi/[(\alpha+\gamma)(\sigma^{-1} - (\alpha+\beta))]^{1/2}$. The values of T derived in the literature are consistent with our interpretation of GPPM.[19] Actually, if the long run motion is due to structural changes, this might also partly explain the observed irregularity of shorter-run cycles, since a shift of the centre implies a change in T. Finally, as argued in Section 3, this explanation of the empirical evidence seems more satisfactory than the long run interpretation, whereas the plots show no evidence of short run persistent fluctuations characterised by the same centre.

From this perspective, unlike for structurally stable models, the interest for GPPM lies *both* in the description in vitro of the basic forces underlying distributive conflict *and* in the fragility of the structure of this interaction. This allows for a less reductionist interpretation of the basic mechanism underlying class conflict and it forcefully stresses the importance of an understanding of structural perturbations. Formally, it puts at the centre of the stage the analysis of the changes in the dynamic features of the model, and thus in the structure of the interaction between classes, arising from structural perturbations.

From a theoretical viewpoint, the proposed interpretation suggests that the extensions of GPPM can be read not only as proofs of its structural instability or as alternative models of distributive conflict. They can be also seen as theoretical analyses of the factors that determine the fragility of the basic mechanism underlying the interaction between classes and thus as an essential part of a full understanding of Marx/Goodwin cycles. While most contributions focus on the existence of cycles in the perturbed models, the potential structural changes are richer and more interesting both in theoretical and in analytical terms. Mathematically, they involve issues like the existence and multiplicity of the equilibria and their dynamic properties. From the viewpoint of economic analysis, the perturbed models describe different economic and institutional environments with different implications for the outcome of distributive conflict.

For instance, in GPPM wages grow on average at the same rate as productivity, so that "we have a Malthusian Iron Law of Profits... [whereby labour is] the sole ultimate beneficiary from technical progress" (Goodwin, 1967, p. 170). Hence, from a Marxist perspective, there is a strong incentive for capitalists to modify the "rules of the game" exploiting the asymmetric position in the production process and their stronger economic and political power. Desai (1973) shows that the introduction of money wage bargaining and inflation has a stabilising effect, yielding dampened cycles around the equilibrium. Moreover, "the presence of inflation means that to get an equivalent rise in real wage rate in the money wage bargaining situation, a higher level of employment (a higher bargaining strength) is necessary" (Desai, 1973, p. 533). Hence Desai

[18] An anonymous referee has suggested that the short-run interpretation is further strengthened if one considers the fact that labour share dynamics can be derived from wage-price dynamics, which describe fairly short run fluctuations in inflation rates (see also Vercelli, 1977). This is an interesting issue for further research.

[19] In general, the values of T derived are considerably lower than the long-run view would suggest: Atkinson (1969) suggests a value of T between 9.8 and 21.9; van der Ploeg (1983) between 10 and 16; and Blatt (1983, p. 210) and Solow (1990, p. 40) between 8 and 10 years. Dore (1993) and Aguiar (2001) compute a value of T around 4.

argues that "inflation is... a way of redistributing income away from labor" (Desai, 1973, p. 533), a result that is strengthened by the introduction of a flexible capital/output ratio.[20]

Desai and Shah (1981) show that if capitalists can also choose the direction of technical change, the steady state is unique and locally asymptotically stable. "Unlike in GPPM, capitalists learn from history in this model. They invest all their profits but they also choose the new technology that will be cost minimising... Any small deviation from equilibrium will allow the capitalists to claw back gains made by workers... At the heart of this result lies the importance of allowing an extra degree of freedom to one party in a model of class struggle" (Desai and Shah, 1981, pp. 1008–1009). The steady state values of u and v are entirely determined, in the last instance, by the capitalists' cost minimisation decisions. An extra degree of freedom yields a change both in the stability properties of the equilibrium and in its value.[21]

Similar results are obtained by van der Ploeg (1983); "fear of redundancies, money illusion and adaptive expectations, under both the cost-push and the quantity theory of inflation, tend to dampen class conflict. On the other hand, the hoarding of labour and the effect of the expected return on capital on the capital–output ratio give rise to exploding cycles of conflict... When one allows for substitution between the factors of production, or for the optimal choice of the direction of technical progress, conflict is completely eliminated" (van der Ploeg, 1983, p. 277).

Another important set of structural perturbations are those affecting the institutional framework in which distributive conflict takes place. Although the government is in the background in GPPM, in a Marxist perspective it is not a neutral actor in class conflict. Indeed, economic policies – monetary (Di Matteo, 1984; Asada, 1989) and fiscal policies (e.g. in the form of unemployment benefits, as in Glombowski and Kruger, 1984) – affect the dynamics of distributive cycles and, more importantly, their position, yielding shifts in the centre.

The above examples show the range of possible structural changes and the economic and mathematical issues that can be analysed by perturbing the model. If the interpretation of GPPM proposed in this paper is correct, these kinds of extensions (especially those that remain in a neighbourhood of the original model) should not be seen as alternative formalisations of distributive conflict, but rather as part of the understanding of distributive conflict, and in particular of the complex interaction between the basic in vitro mechanism underlying class struggle depicted by GPPM and the endogenous forces that tend to modify it. Of course, this suggests a more open-ended approach to GPPM – and to model building in general – but this is consistent with Goodwin's own methodological approach, whereby "given that history does not repeat itself, the formal business cycle theory cannot contain all the truth" (Goodwin, 1982, p. 18).

5. Conclusions

In this paper, the methodological prescription that a structurally unstable model should be rejected is analysed by considering the specific case of Goodwin's predator–prey model (GPPM; Goodwin, 1967, 1972). The inherent ambiguities in the definition of structural instability and the difficulties of proving the structural instability of GPPM are highlighted. It is argued that structural instability is not sufficient to reject a model a priori and that alternative models should

[20] For a thorough formal analysis of Desai (1973), which emphasises structural changes (including some not analysed by Desai), see Veneziani (2001).

[21] The complex relation between distributive conflict and technical change in a Marxian framework is discussed in more detail in Mohun and Veneziani (2006).

be evaluated based on the empirical features of the phenomenon investigated and on the aims of the research. GPPM is compared with its structurally stable extensions yielding limit-cycles, taking into account both the empirical evidence and the main features of the theory that underlie the model. No general methodological prescriptions on the desirable stability properties of models of class based societies are advanced, yet, it is argued that from both points of view, structurally stable models do not necessarily represent more satisfactory formalisations of Marx's theory of distributive conflict.

An interpretation of GPPM is proposed which takes account of its structural instability. It is a theoretical model to understand distributive conflict, rather than a self-contained description either of a long run cycle or of a short-run persistent cycle. It is a stylised, in vitro portrait of the basic forces underlying class conflict and its structural instability is an extreme picture of the fragility of the structure of the symbiotic mechanism regulating distributive conflict.

From an empirical viewpoint, this suggests identifying Goodwin's cycles with the shorter-run cycles visible in the uv-plots around a long run motion, while the latter should be taken as the product of structural change. The analysis of the post-war US data lends considerable support to this interpretation (Mohun and Veneziani, 2006) and it raises some interesting directions for further empirical research. From a theoretical viewpoint, the proposed interpretation suggests an open-ended approach to GPPM that emphasises the importance of understanding structural perturbations, including discontinuous structural change. From this perspective, several extensions of the model are interpreted as part of the understanding of distributive conflict, and in particular of the complex interaction between the basic in vitro mechanism underlying class struggle depicted by GPPM and the endogenous forces that tend to modify it.

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